Relating Perimeter and Area
Explore Themes in Student Thinking

Throughout the video, students express a range of mathematical ideas about the relationship between perimeter and area. Students talk about the relationship between squaring and multiplication, between dimension and perimeter, and dimension and area. While clearly related these three themes each highlight an essential aspect of the relationship between perimeter and area.

Theme 1: Relationship between Squaring and Multiplication
Although Julia introduces relationship between changes in perimeter and area in terms of "squaring," students describe the rule in terms of multiplication, exponents, and squaring. For many students the relationship between squaring (or more generally exponents) and multiplication seems unproblematic – they seamlessly talk about one or the others without difficulty. For example, Riaz very clearly states his understanding of the relationship between squaring and multiplication by saying, "And so, um, you have to square it because it's this number times this number, which is essentially squaring." Other students are less explicit about the connection. Jake first describes the rule as "you're gonna have to multiply that number by itself." He goes on to say "you have to square the total" by "multiplying it five times by five times." In his exchange with Monica she repeatedly restates the rule using multiplication: "multiply each dimension by five times" and "five times by five times."

Other students' language suggests some confusion, though it is not clear whether this confusion is a deep conceptual issue or just a slip of the tongue. Evidence of at least momentary confusion over the relationship between squaring and multiplication comes from a discussion between Cory, Riaz, and the teacher at the start of the clip. Cory restates the class' rule as "three to the power of itself" or "three with the little exponent of three." The teacher suggests that she means "three times itself" and Riaz suggests that she means "three squared." Cory quickly changes her language to agree with Riaz's "three squared" idea. The teacher then repeats for the class the relationship between squaring and multiplication: "So three times itself or three to the power of two."

Theme 2: Relationship between Dimension and Perimeter
The question posed to the class asks for the relationship between perimeter and area. However, several students talk about the generalized relationship between dimensions and area rather than perimeter and area. For example, Jake focuses on finding the area from the dimensions, "you find the area by using two dimensions for the triangle; it's the base and the height. And for the rectangle it's the length and the width." Similarly, Riaz uses hand motions to indicate the length and width increasing when talking about the increase in area. Coleman also focuses on dimensions when he explains his visual representation of the relationship, "when we double the length, like so, and double the width, like so, um, we basically create three more squares."

It may be the case that the relationship between dimension and perimeter is obvious for this class, such that the tacit assumption underlying what students said is that when the dimensions are increased by a certain factor, the perimeter is also increased by the same factor. The only student who may be expressing confusion about the relationship between dimension and perimeter in the

clip is Monica when she says, “so you’re saying you have to multiply each dimension by five times?” Her initial confusion and her difficulty to “spit it out” when she “get’s it” may indicate that she is not completely comfortable moving between dimensions and perimeter.

Theme 3: Relationship between Dimension and Area

Much of the student discussion about the relationship between perimeter and area centers around the relationship between dimension and area. Jake focuses on using the dimensions to determine the area because he says, “for both of the shapes you're looking at, uh, you find the area by using two dimensions.” He then connects the multiplication of the dimensions by a certain factor to the resulting area by saying, “when you multiply each of those dimensions by a certain number you're gonna have to multiply, you’re gonna have to multiply that number by itself because you’re using a, both of those dimensions.” Riaz demonstrates this relationship between dimensions and area physically when he indicates how the length and width are increasing with his hands.

Coleman uses a visual representation to illustrate that doubling the sides of a square results in the creation of four total squares, which quadruples the area. Coleman explains his reasoning saying, “when we double the length, like so, and double the width, like so, um, we basically create three more squares and thus have a square that’s quadruple the area of the previous one.” Therefore, Coleman focuses on the additional squares or area created by doubling the dimensions and therefore highlights the relationship between the original area and the resulting area by using the dimensions.