Sequences
Explore Themes in Student Thinking

Throughout the video, students describe their reasoning about how to find the heights of towers in a sequence. Students use two representations to model the mathematical relationships described in the problem: manipulatives, specifically centimeter cubes, and symbolic notation.

Theme 1: Modeling Mathematical Relationships with Manipulatives
In this video, students begin with a candidate formula for the height of the tallest tower and use the cubes to explore the mathematical relationships. For example, at the start of the discussion Allison uses the towers of cubes they have built to justify her formula. She first points out that the formula $h_n=nh_1$ works for the first tower in the sequence; "See, like for this one [the first] it [the height] is two." She then counts the "distance" to the fifth tower (n=5) and the number of cubes in the fifth tower (h_5=10) to illustrate that the formula holds for that tower also (10=5*2).

Allison and Jen also work with the cubes to test the conjecture that their formula will work when the difference is a value other than two. Jen suggests changing the difference to three, and Allison adds one cube to each tower to test this idea. Jen points out that adding one cube will not correctly model a mathematical difference of three; Allison instead needs to add "one [more] here" to each tower. Here, Allison and Jen create new towers to test the adequacy of their formula with other possible values.

The previous examples suggest that modeling mathematical relationships with manipulatives can be useful in mathematics because it allows students to justify and test formulas. However, doing so can also create some difficulties. For example, after Allison has completed making a difference of three, she and Jen count the cubes and find a discrepancy between the formula and the towers. To resolve this discrepancy, Jen seeks a way to "fix" the towers without changing their formula. She suggests that if they make the first tower three, then "three times five is still fifteen [the calculated value from the formula.]" Rather than the cubes indicating that they need to reconsider the original formula, the students take the mismatch between the formula and the manipulatives to be an issue with the towers. In this case, modeling with manipulatives does not assist students in making progress in refining their formula.

Theme 2: Modeling Mathematical Relationships with Symbols
In the video students are working to find a formula the height of the tallest tower. A critical first step in constructing any formula is to identify the relevant variables. Which variables in the problem situation should be put into the formula? From the start of their discussion, students seem to accept Allison's suggestion that they include "the place it [the tower] is in line" or "the number of the distance" (n); no other students contest the relevance of this variable. Allison also suggests they include "the height of the original (h_1); Carol echoes this idea saying, "so it's really the height of the first one."

However, the students do not agree on whether to include the difference (d) in their formula. Allison states, "I don't think the difference [matters]" but Jen is surprised by Allison's decision to leave the difference out. She says, "The difference wouldn't work you don't think?" Here, Allison and Jen seem to disagree about whether the difference matters – whether *d* should be an variable in the formula.

At other times students seem to believe that *d* matters, but they also assume that it is equivalent to the height of the first tower and can thus be excluded from the formula. Early on Jen questions this assumption, asking whether Allison's formula ($h_n = nh_1$) would work "if it [the difference] wasn't necessarily the height of the first [tower.]" Later however, in response to why the formula is not working Jen states "this [the first tower] has to be three [same as the difference];" she makes the very assumption she questioned at the start. At the end of the video Allison reposes the question, "Wait, does the height of the first one have to equal the difference between all of them?" Part of the students' difficulty in deciding on a formula for h_n involving n, d, and h_1 may lie in their unexamined assumption that $d=h_1$. They have not articulated the three possibilities at play: d is irrelevant, d is relevant but equal to h_1, or d is relevant and not equal to h_1.